Math 116
Spring 2017

Study Guide for Exam 1

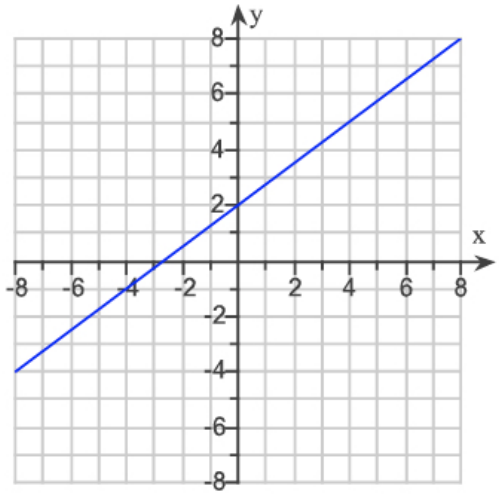
1. Find the slope of a line that goes through the points $(-3, 5)$ and $\left(1, \frac{1}{2}\right)$.
   The slope is

   (A) between -2 and $-\frac{3}{2}$ **(B)** between $-\frac{3}{2}$ and -1 **(C)** between -1 and $-\frac{1}{2}$

   (D) between $-\frac{1}{2}$ and 0 **(E)** undefined

   FOLLOW-UP: *Be sure you can recognize when the slope is undefined, or is zero.*

2. Estimate the slope of the line given below.



   The slope is

   (A) between 0 and $\frac{1}{2}$ **(B)** between $\frac{1}{2}$ and 1 **(C)** between 1 and $\frac{3}{2}$

   (D) between $\frac{3}{2}$ and 2 **(E)** undefined

   FOLLOW-UP: *Be sure you can recognize when the slope is undefined, or is zero.*

3. Determine the equation of the line that goes through the point $\left(-5, -\frac{7}{2}\right)$ with slope $-\frac{1}{2}$.

(A) $y = -\frac{1}{2}x - 6$ **(B)** $y = -\frac{1}{2}x + 1$ **(C)** $y = -\frac{1}{2}x - \frac{7}{2}$

(D) $y = -5x - \frac{7}{2}$ **(E)** None of these

FOLLOW-UP: *Be sure you can do this when the slope is undefined, or is zero.*

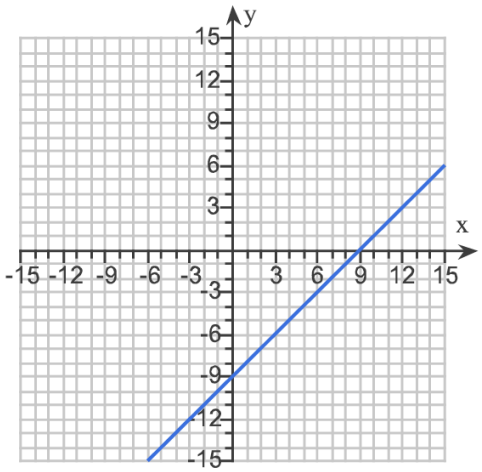
4. Determine the equation of the line that goes through the points $\left(-2,\ \frac{3}{4}\right)$ and $\left(\frac{2}{3},\ \frac{5}{2}\right)$.

(A) $y = \frac{14}{3}x + \frac{121}{12}$ **(B)** $y = \frac{14}{3}x + \frac{3}{4}$ **(C)** $y = \frac{21}{32}x + \frac{5}{2}$

(D) $y = \frac{21}{32}x + \frac{33}{16}$ **(E)** None of these

FOLLOW-UP: *Be sure you can do this when the slope is undefined, or is zero.*

5. Determine the equation of the line given below.



(A) $y = x - 9$ **(B)** $y = 9x - 9$ **(C)** $y = x - 1$

(D) $y = 9x - 1$ **(E)** None of these

FOLLOW-UP: *Be sure you can determine the equation of a vertical or horizontal line as well.*

6. World grain production was 1241 million tons in 1975 and 2048 million tons in 2005, and has been increasing at an approximately constant rate. Answer parts **(i)** through **(iii)**.

(i) Determine a linear equation that approximates world grain production, P, in millions of tons, in terms of t, the number of years since 1975.

(A) $P = 26.9t = 54368.5$ **(B)** $t = 0.037t + 1241$ **(C)** $P = 1241t + 2048$

(D) $P = 26.9t + 1241$ **(E)** None of these

(ii) Using units, interpret the slope in terms of grain production.

(iii) According to the linear model, when is grain production predicted to reach 2500 million tons?

7. A ski resort charges a snowboard rental fee of \$20 plus \$9.25 per hour.
   Answer parts **(i)** through **(iii)**.

   (i) Write a linear cost function $C(t)$, where t is the time in hours, for the following situation.

   (A) $C(t) = 20t + 9.25$ **(B)** $C(t) = 9.25t + 20$ **(C)** $C(t) = 20t + 29.25$

   (D) $C(t) = 9.25t + 29.95$ **(E)** None of these

   (ii) Find $C(5)$.

   (iii) Interpret $C(5)$ in the context of the problem.

8. The Dispatch Tool Works spends \$9000 to produce 130 parts, achieving a marginal cost of \$60. Find the linear cost function $C(x)$, where x is the number of parts produced.

(A) $C(x) = 130x + 60$ **(B)** $C(x) = 130x + 9000$ **(C)** $C(x) = 60x + 1200$

(D) $C(x) = 60x + 9000$ **(E)** None of these

9. The manager of a restaurant found that the cost to produce 300 cups of coffee is \$68.85, while the cost to produce 700 cups is \$116.85. Assume the cost $C(x)$ is a linear function of x, the number of cups produced. Answer parts **(i)** through **(iii)**.

(i) Find the total cost of producing 1200 cups. The total cost is

(A) between \$160 and \$170

(B) between \$170 and \$180

(C) between \$180 and \$190

(D) between \$190 and \$200

(E) between \$200 and \$210

(ii) Find the marginal cost of a cup of coffee. The marginal cost is

(A) between \$0.10 and \$0.15

(B) between \$0.15 and \$0.20

(C) between \$0.20 and \$0.25

(D) between \$0.25 and \$0.30

(E) between \$0.30 and \$0.35

(iii) What does the marginal cost of a cup of coffee mean to the manager?

(A) The marginal cost of a cup of coffee is the cost of producing a given number of cups.

(B) The marginal cost of a cup of coffee is the cost of producing the first cup.

(C) The marginal cost of a cup of coffee is the cost of producing one additional cup.

(D) The marginal cost of a cup of coffee is the cost of producing zero cups.

(E) None of these.

10. A product has a production cost function $C(x) = 460x + 6670$ and a revenue function $R(x) = 575x$. Find the break-even quantity.

    The break-even quantity is

    (A) between 20 units and 30 units

    (B) between 30 units and 40 units

    (C) between 40 units and 50 units

    (D) between 50 units and 60 units

    (E) between 60 units and 70 units

11. Joanne sells silk-screened T-shirts at community festivals and craft fairs. Her marginal cost to produce one T-shirt is \$3.50. Her total cost to produce 42 T-shirts is \$245, and she sells them for \$7 each. How many T-shirts must she produce and sell in order to break even?

    To break even, Joanne must produce and sell

    (A) less than 25 T-shirts

    (B) between 25 and 30 T-shirts

    (C) between 30 and 35 T-shirts

    (D) between 35 and 40 T-shirts

    (E) more than 40 T-shirts

12. Let one week's supply and demand functions for gasoline be given by

$$p = D(q) = 300 - \frac{2}{5}q \quad \text{and} \quad p = S(q) = \frac{4}{5}q,$$

where p is the price in dollars and q is the number of 42-gallon barrels. Answer parts **(i)** and **(ii)** below.

(i) Graph these equations on the same axes.

(ii) Find the equilibrium price.

13. Find the domain of the function $f(x) = \dfrac{3}{x^2 - 4}$.

(A) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ **(B)** $(-\infty, \infty)$ **(C)** $(2, \infty)$

(D) $(-\infty, 0) \cup (0, \infty)$ **(E)** None of these

14. Let

$$f(x) = \begin{cases} \dfrac{2x+2}{x-9} & \text{if } x \neq 9 \\ 2 & \text{if } x = 9 \end{cases}$$

Answer parts **(i)** through **(iii)**

(i) Find the value of $f(9)$. The value of $f(9)$ is

(A) less than -1 **(B)** between -1 and 1 **(C)** between 1 and 3

(D) between 3 and 5 **(E)** more than 5

(ii) Find the value of $f(-\frac{1}{2})$. The value of $f(-\frac{1}{2})$ is

(A) between -3 and -1 **(B)** between -1 and 1 **(C)** between 1 and 3

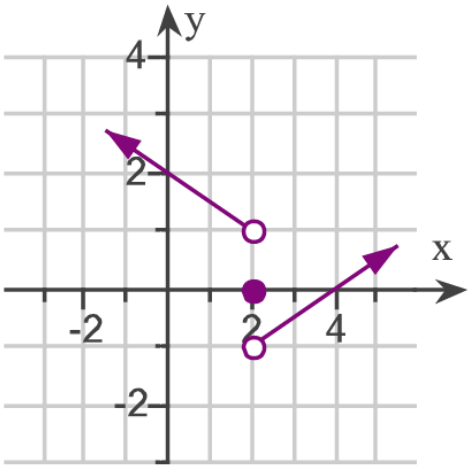
(D) between 3 and 5 **(E)** between 5 and 7

(iii) Find the value of x such that $f(x) = 1$. The value of x is

(A) less than -5 **(B)** between -5 and 5 **(C)** between 5 and 15

(D) between 15 and 25 **(E)** more than 25

15. The graph of $g(x)$ is given below



Answer parts **(i)** through **(iii)**

(i) Find the value of $g(2)$.

(A) -1 **(B)** 0 **(C)** 1 **(D)** 4 **(E)** None of these

(ii) Find the slope of $g(x)$ at $x = 1$.

(A) $-\frac{1}{2}$ **(B)** -1 **(C)** 0 **(D)** $\frac{1}{2}$ **(E)** None of these

(iii) Find the value(s) of x such that $g(x) = 0$.

16. Income tax brackets often take the form of piecewise linear functions. Consider the 2010 U.S. Rate Schedule for single persons, showing the income tax owed, T, as a function of adjusted income, i.

$$T = \begin{cases} 0.10i & \text{for } 0 \le i \le 8,375 \\ 0.15(i - 8375) + 837.50 & \text{for } 8,375 < i \le 34,000 \\ 0.25(i - 34000) + 4,681.25 & \text{for } i > 34,000 \end{cases}$$

(i) If a single person earns an adjusted income of \$30,000, how much income tax would the person owe?

The person would owe

(A) between \$2,900 and \$3,200 **(B)** between \$3,200 and \$3,500

(C) between \$3,500 and \$3,800 **(D)** between \$3,800 and \$4,100

(E) between \$4,100 and \$4,400

(ii) If a person owes \$5000 in income tax, what was their adjusted income?

Their adjusted income was

(A) between \$32,000 and \$36,000 **(B)** between \$36,000 and \$40,000

(C) between \$40,000 and \$44,000 **(D)** between \$44,000 and \$48,000

(E) between \$48,000 and \$52,000

17. Consider the parabola

$$y = 2x^2 + 8x - 10.$$

Answer parts **(i)** through **(v)**

(i) Find the vertex of the parabola. The x-coordinate of the vertex is

(A) between -4 and -1 **(B)** between -1 and 2 **(C)** between 2 and 5

(D) between 5 and 8 **(E)** between 8 and 11

(ii) Find the vertex of the parabola. The y-coordinate of the vertex is

(A) between -20 and -17 **(B)** between -17 and -14 **(C)** between -14 and -11

(D) between -11 and -8 **(E)** between -8 and -5

(iii) The vertex is

(A) a maximum **(B)** a minimum **(C)** neither a max nor a min

(iv) Find the y-intercept of the parabola. The y-intercept is

(A) between -20 and -17 **(B)** between -17 and -14 **(C)** between -14 and -11

(D) between -11 and -8 **(E)** between -8 and -5

(v) Find the x-intercepts of the parabola. The larger of the x-intercepts is

(A) between -3 and 0 **(B)** between 0 and 3 **(C)** between 3 and 6

(D) between 6 and 9 **(E)** between 9 and 12

18. Let $C(x) = 3x + 35$ be the cost, in dollars, to produce x widgets, and let $R(x) = -x^2 + 15x$ be the revenue, in dollars, from selling x widgets. Answer parts **(i)** through **(ii)**

(i) Find the minimum break-even quantity. The minimum break-even quantity is

(A) 6 **(B)** 5 **(C)** 7.5 **(D)** 7 **(E)** none of these

(ii) Find the maximum profit. The maximum profit is

(A) \$7.50 **(B)** \$56 **(C)** \$1 **(D)** \$6 **(E)** none of these

19. The manager of a 60-unit apartment complex is trying to decide what rent to charge. Experience has shown that at a rent of \$900, all the units will be full. On the average, one additional unit will remain vacant for each \$50 increase in rent.

(i) Let x represent the number of \$50 increases. Find an expression for the total revenue from all rented apartments.

(ii) What value of x leads to maximum revenue?

(iii) What is the maximum revenue?

20. A charter boat charges a fare of \$120 per person plus \$3 for each unsold seat on the boat. The boat holds 80 passengers. Let x represent the number of unsold seats. Answer parts **(i)** and **(ii)**

(i) Find an expression for the total revenue received for the boat.

(A) Revenue $= 9600 + 3x$

(B) Revenue $= (120 + 3x)(80 - x)$

(C) Revenue $= 80(120 + 3x)$

(D) Revenue $= 120x + 3x(80 - x)$

(E) None of these

(ii) Find the maximum revenue. The maximum revenue is

(A) between \$8,000 and \$9,000

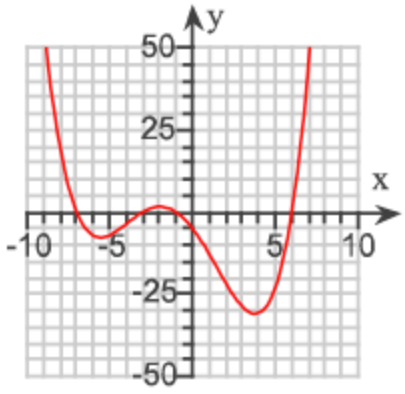
(B) between \$9,000 and \$10,000

(C) between \$10,000 and \$11,000

(D) between \$11,000 and \$12,000

(E) between \$12,000 and \$13,000

21. The graph below is the graph of a polynomial. Give the possible degree of the polynomial, and give the sign (positive or negative) of the leading coefficient.



(A) Degree 4 with negative leading coefficient

(B) Degree 4 with positive leading coefficient

(C) Degree 5 with negative leading coefficient

(D) Degree 5 with positive leading coefficient

(E) None of the above

22. Let $y = \dfrac{3-2x}{2x+17}$. Answer parts **(i)** through **(iii)**

(i) What is the vertical asymptote?

(A) $x = -1$ **(B)** $x = -\dfrac{17}{2}$ **(C)** $y = \dfrac{3}{2}$

(D) $y = -1$ **(E)** There is no vertical asymptote

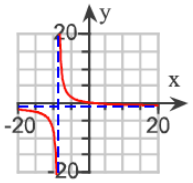
(ii) What is the horizontal asymptote?

(A) $x = -1$ **(B)** $x = -\dfrac{17}{2}$ **(C)** $y = \dfrac{3}{2}$

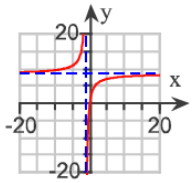
(D) $y = -1$ **(E)** There is no horizontal asymptote

(iii) Graph the function. Choose the correct graph below.

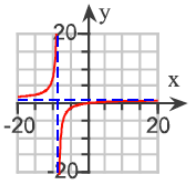
A.



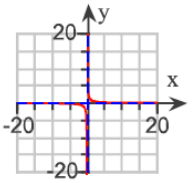
B.



C.



D.



23. Suppose a cost-benefit model is given by

$$y = \frac{6.6x}{100 - x}$$

where x is a number of percent and y is the cost, in thousands of dollars, of removing x percent of a given pollutant.

(i) Find the cost of removing 95% of the given pollutant.

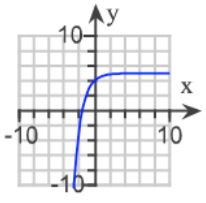
(A) \$125,400 **(B)** \$125.40 **(C)** \$6,330 **(D)** \$0.06 **(E)** None of these

(ii) Is it possible, according to this function, to remove all the pollutant?

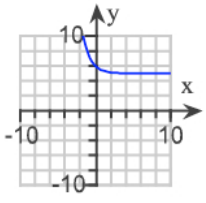
(iii) Graph the function.

24. Graph the function $f(x) = 5 - 2^{-x}$. Choose the correct graph.

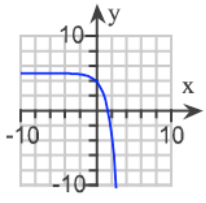
○ A.



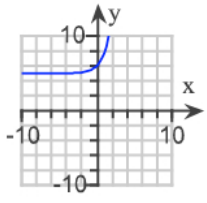
○ B.



○ C.



○ D.



25. Solve for x:

$$e^{-3x} = (e^8)^{2-x}.$$

x is

(A) between 1 and 2 **(B)** between 2 and 3 **(C)** between 3 and 4

(D) between 4 and 5 **(E)** between 5 and 6

26. Leigh Jacks plans to invest $6,000 into a money market account. Find the interest rate that is needed for the money to grow to $45,000 in 30 years if the interest is compounded quarterly.

The interest rate is

(A) between 6.65% and 6.75%

(B) between 6.75% and 6.85%

(C) between 6.85% and 6.95%

(D) between 6.95% and 7.05%

(E) between 7.05% and 7.15%

27. An item costs $5000 today. What will the item cost in 6 years assuming an inflation rate of 4.2% compounded continuously?

It will cost

(A) between $6424 and $6426

(B) between $6426 and $6428

(C) between $6428 and $6430

(D) between $6430 and $6432

(E) between $6432 and $6434

28. Find the interest earned on \$25,000 invested for 3 years at 4% interest compounded as follows. (Round your answer to the nearest cent.)

(i) Annually:

(A) \$3,000.00 **(B)** \$3,121.60 **(C)** \$3,181.80 **(D)** \$3,187.42 **(E)** \$28,121.60

(ii) Monthly:

(A) \$3,000.00 **(B)** \$3,121.60 **(C)** \$3,181.80 **(D)** \$3,187.42 **(E)** \$28,181.80

(iii) Continuously:

(A) \$3,000.00 **(B)** \$3,121.60 **(C)** \$3,181.80 **(D)** \$3,187.42 **(E)** \$28,187.42

29. Use natural logarithms to evaluate $\log_6 50$.

$\log_6 50$ is

(A) between 1 and 2 **(B)** between 2 and 3 **(C)** between 3 and 4

(D) between 4 and 5 **(E)** between 5 and 6

30. Solve for x:

$$3^{2x-1} = 187.$$

x is

(A) between 2 and 2.5 **(B)** between 2.5 and 3 **(C)** between 3 and 3.5

(D) between 3.5 and 4 **(E)** between 4 and 4.5

31. May Klingman invests \$15,000 in an account paying 3% per year, compounded quarterly. How many years are required for the compound amount to at least double?

The time it takes for the compound amount to at least double is

(A) less than 5 years **(B)** between 5 and 10 years **(C)** between 10 and 15 years

(D) between 15 and 20 years **(E)** more than 20 years

32. What is the difference between stated interest rate and effective rate? Choose the correct answer below.

(A) The stated interest rate is the rate used to calculate future value based on the present value. The effective interest rate is the rate used to calculate present value based on future value.

(B) The stated interest rate is the percentage per compounding period. The effective interest rate is the stated interest rate multiplied by the number of compounding periods.

(C) The stated interest rate is the annual interest rate before adjusting for compounding. The effective interest rate is the actual percentage increase after compounding.

(D) The stated interest rate is the annual yield. The effective interest rate is the yield from continuous compounding.

33. Find the effective rate corresponding to a nominal rate of 8.35% compounded continuously. (Round your answer to the nearest hundredth of a percent.)

The effective rate is

(A) 8.35% **(B)** 8.62% **(C)** 8.68% **(D)** 8.71% **(E)** none of these

FOLLOW-UP: *Be sure you can find the effective rate if the rate is compounded quarterly, monthly, daily, etc...*

34. Frank Steek must make a balloon payment of $25,000 in 7 years. Find the present value of the payment if it includes annual interest of 5.2% compounded monthly.

The present value is

(A) less than $17,350

(B) between $17,350 and $17,375

(C) between $17,375 and $17,400

(D) between $17,400 and $17,425

(E) more than $17,425

35. Southwest Dry Cleaners believes that it will need new equipment in 11 years. The equipment will cost $175,000. What lump sum should be invested today at 4.21% compounded continuously, to yield $175,000?

The lump sum is

(A) between $109,800 and $109,900

(B) between $109,900 and $110,000

(C) between $110,000 and $110,100

(D) between $110,100 and $110,200

(E) between $110,200 and $110,300

36. Christine O'Brien, who is self-employed, wants to invest \$80,000 in a pension plan. One investment offers 6% compounded quarterly. Another offers 5.75% compounded continuously.

(i) Which investment will earn the most interest in 4 years?

(ii) How much more will the better plan earn?

(iii) What is the effective rate in each case?

(iv) If Ms. O'Brien chooses the plan with continuous compounding, how long will it take for her \$80,000 to grow to \$90,000? (Round to two decimal places.)

37. Sales of a new model of compact disc player are approximated by the function

$$S(x) = 1100 - 800e^{-x},$$

where $S(x)$ is in appropriate units and x represents the number of years the disc player has been on the market

(i) Find the sales during year 0.

(ii) In how many years will sales reach 900 units? (Round to two decimal places.)

(iii) Will sales ever reach 1,100 units?

(iv) Is there a limit on sales for this product? If so, what is it?

38. Use the table of values to answer parts **(i)** and **(ii)**.

x	6.9	6.99	6.999	6.9999	7	7.0001	7.001	7.01
$f(x)$	9.9	9.99	9.999	9.9999	18	10.0001	10.001	10.01

(i) Estimate $\lim_{x\to 7} f(x)$.

(A) 7 **(B)** 10 **(C)** 9.9 **(D)** 18 **(E)** the limit does not exist

(ii) Evaluate $f(7)$.

(A) 7 **(B)** 10 **(C)** 9.9 **(D)** 18 **(E)** the value does not exist

39. Suppose $\lim_{x\to 5^-} f(x) = 9$, and $\lim_{x\to 5^+} f(x) = 9$, but $f(5)$ does not exist. What can you say about $\lim_{x\to 5} f(x)$?

(A) $\lim_{x\to 5} f(x) = 9$ **(B)** $\lim_{x\to 5} f(x) = -9$ **(C)** $\lim_{x\to 5} f(x)$ does not exist

(D) $\lim_{x\to 5} f(x) = \infty$ **(E)** None of these

40. Let $f(x) = \dfrac{x^2-1}{x+1}$. Answer parts **(i)** through **(iii)**

(i) Complete the table below.

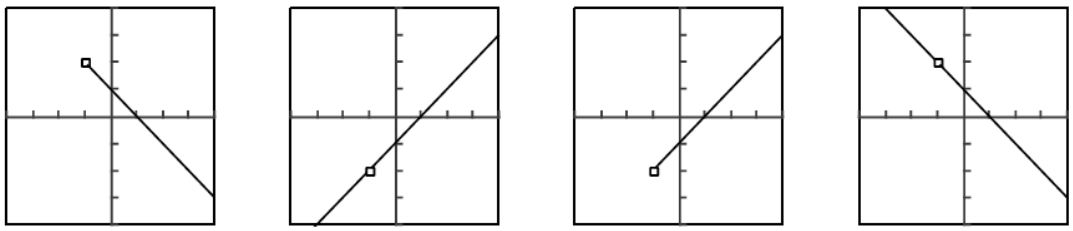
x	-1.1	-1.01	-1.001	-0.999	-0.99	-0.9
$\dfrac{x^2-1}{x+1}$						

(ii) Calculate $\lim\limits_{x\to -1} \dfrac{x^2-1}{x+1}$.

(A) -2 **(B)** -1 **(C)** ∞ **(D)** 2 **(E)** The limit does not exist

(iii) Verify your answer by using a graphing calculator. Choose the correct graph below. The graph below is displayed on a $[-4, 4, 1]$ by $[-4, 4, 1]$ window.

○ A. ○ B. ○ C. ○ D.



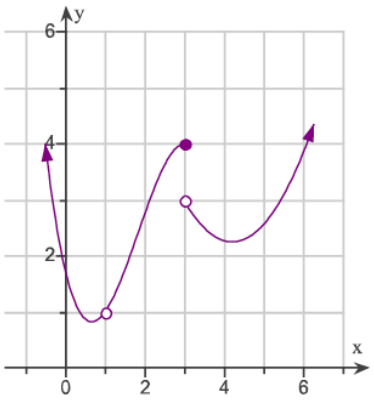
41. Calculate $\lim_{x \to -2} \left(x^2 - x + 4\right)$.

42. Calculate $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

(A) $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3} = 3$ **(B)** $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3} = 6$ **(C)** $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3} = 9$

(D) $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3} = \infty$ **(E)** $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ does not exist

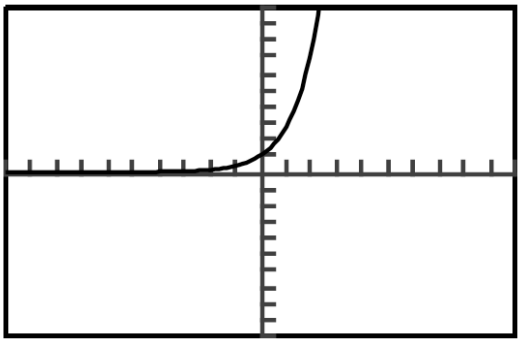
43. The graph of $f(x)$ is given below.



Use the graph to find $\lim_{x \to 3} f(x)$.

(A) 4 **(B)** 3 **(C)** 3.5 **(D)** 0 **(E)** The limit does not exist

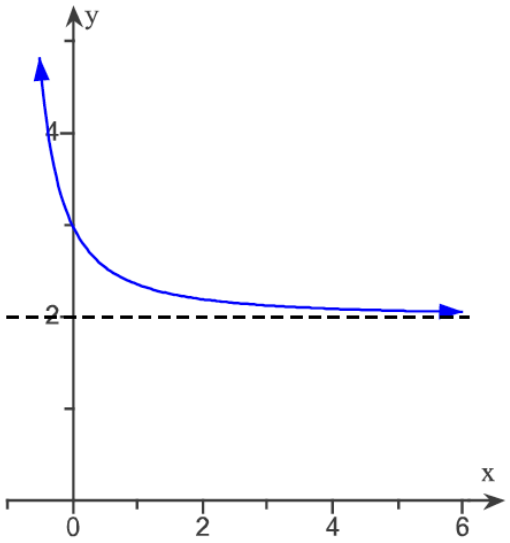
44. Use the graph of $f(x) = e^x$ below to find $\lim_{x \to -\infty} e^x$.



(A) $\lim_{x \to -\infty} e^x = e$ **(B)** $\lim_{x \to -\infty} e^x$ does not exist **(C)** $\lim_{x \to -\infty} e^x = \infty$

(D) $\lim_{x \to -\infty} e^x = 0$ **(E)** None of these

45. The graph of $f(x) = e^{-x} + 2$ is given below.



Use the graph to find $\lim_{x\to\infty} f(x)$, if it exists.

(A) The limit does not exist **(B)** ∞ **(C)** 5 **(D)** 3 **(E)** 2

46. Find $\lim_{x\to\infty} \dfrac{3}{2x-1}$.

(A) $\lim_{x\to\infty} \dfrac{3}{2x-1} = \dfrac{3}{2}$ **(B)** $\lim_{x\to\infty} \dfrac{3}{2x-1}$ does not exist **(C)** $\lim_{x\to\infty} \dfrac{3}{2x-1} = \infty$

(D) $\lim_{x\to\infty} \dfrac{3}{2x-1} = 0$ **(E)** None of these

47. Find $\lim_{x\to\infty} \dfrac{2x+1}{3x-4}$.

(A) $\lim_{x\to\infty} \dfrac{2x+1}{3x-4} = \dfrac{2}{3}$ **(B)** $\lim_{x\to\infty} \dfrac{2x+1}{3x-4}$ does not exist **(C)** $\lim_{x\to\infty} \dfrac{2x+1}{3x-4} = \infty$

(D) $\lim_{x\to\infty} \dfrac{2x+1}{3x-4} = 0$ **(E)** None of these

48. Let $f(x) = 7x^3 + 7$. Answer parts **(i)** through **(iii)**

(i) Find the average rate of change of the function $f(x) = 7x^3 + 7$ over the interval $[5, 7]$.
The average rate of change is

(A) less than 600 **(B)** between 600 and 650 **(C)** between 650 and 700

(D) between 700 and 750 **(E)** more than 750

(ii) Find the average rate of change of the function $f(x) = 7x^3 + 7$ over the interval $[-1, 1]$.
The average rate of change is

(A) less than -10 **(B)** between -10 and 0 **(C)** between 0 and 10

(D) between 10 and 20 **(E)** more than 20

(iii) Find the instantaneous rate of change of the function $f(x) = 7x^3 + 7$ at $x = 5$.
The instananeous rate of change is

(A) less than 600 **(B)** between 600 and 650 **(C)** between 650 and 700

(D) between 700 and 750 **(E)** more than 750

49. Find the instantaneous rate of change of $g(t) = 5 - t^2$ at $t = -5$.

The instantaneous rate of change is

(A) less than -15 **(B)** between -15 and -5 **(C)** between -5 and 5

(D) between 5 and 15 **(E)** more than 15

50. Suppose customers in a hardware store are willing to buy $N(p)$ boxes of nails at p dollars per box, as given by

$$N(p) = 80 - 5p^2; \quad 1 \leq p \leq 4.$$

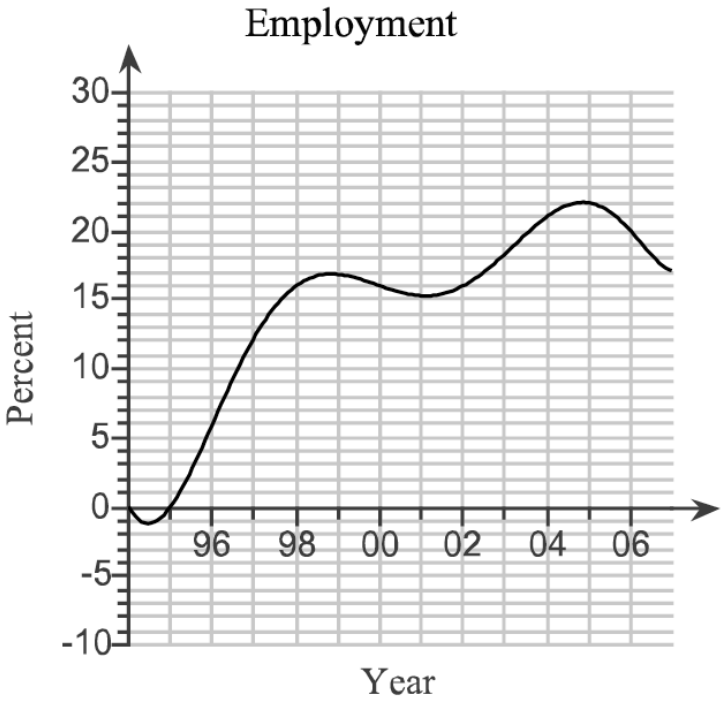
Find the instantaneous rate of change of demand when the price is \$2.

The instantaneous rate of change of demand when the price is \$2 is

(A) less than -15 **(B)** between -15 and -5 **(C)** between -5 and 5

(D) between 5 and 15 **(E)** more than 15

51. Use the graph below to estimate the average rate of change of the percentage of new employees from 2000 to 2006.



The average rate of change is

(A) between 0% and 1% per year

(B) between 1% and 2% per year

(C) between 2% and 3% per year

(D) between 3% and 4% per year

(E) between 4% and 5% per year

52. Suppose that the total profit in hundreds of dollars from selling x items is given by $P(x) = 2x^2 - 7x + 5$. Answer parts **(i)** through **(iii)**

(i) Find the average rate of change of profit as x changes from 3 to 5.
The average rate of change is

(A) less than \$650 per item

(B) between \$650 and \$750 per item

(C) between \$750 and \$850 per item

(D) between \$850 and \$950 per item

(E) more than \$950 per item

(ii) Find and interpret the instantaneous rate of change of profit with respect to the number of items produced when $x = 3$. (This number is called the marginal profit at $x = 3$.)

(A) When items are sold for \$500, the profit is decreasing at the rate of \$3 per item.

(B) When items are sold for \$500, the profit is increasing at the rate of \$3 per item.

(C) When 3 items are sold, the profit is increasing at the rate of \$500 per item.

(D) When 3 items are sold, the profit is decreasing at the rate of \$500 per item.

(E) None of these.

(iii) Find the marginal profit when 5 items are sold.
The marginal profit is

(A) less than \$650 per item

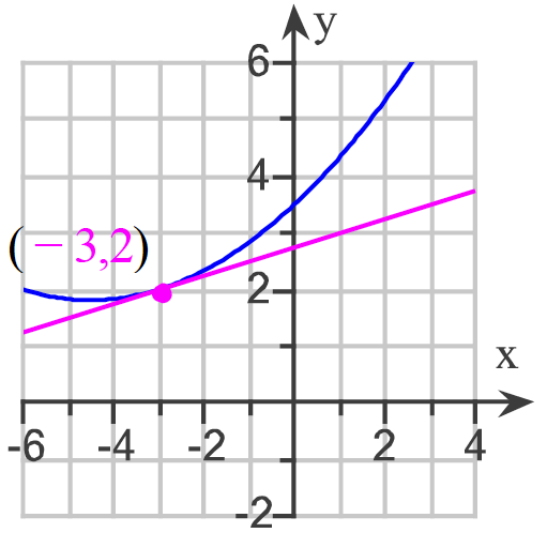
(B) between \$650 and \$750 per item

(C) between \$750 and \$850 per item

(D) between \$850 and \$950 per item

(E) more than \$950 per item

53. Estimate the slope of the tangent line to the curve at the point $(-3, 2)$.



The slope is

(A) less than $-\frac{1}{2}$ **(B)** between $-\frac{1}{2}$ and 0 **(C)** between 0 and $\frac{1}{2}$

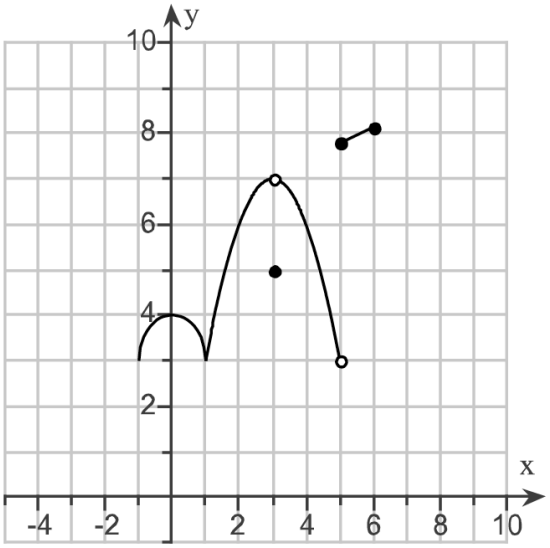
(D) between $\frac{1}{2}$ and 1 **(E)** more than 1

54. Find $f'(3)$ for the function $f(x) = 2e^x$, if the derivative exists.
The value of $f'(3)$ is

(A) less than 35 **(B)** between 35 and 45 **(C)** between 45 and 55

(D) more than 55 **(E)** does not exist

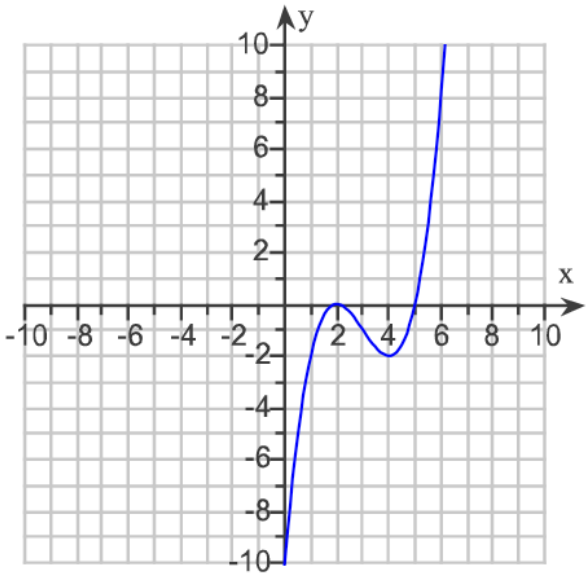
55. List the points in the graph in the interval $-1 < x < 6$ at which the function is not differentiable.



(A) $x = 1$ only **(B)** $x = 1$, $x = 3$, $x = 5$ **(C)** $x = 1$, $x = 3$ only

(D) $x = 5$ only **(E)** None of these

56. For the function shown in the graph below, answer parts **(i)** through **(iii)**



(i) Choose the interval(s) on which the *rate of change* is positive.

(A) $(-\infty, 2)$ and $(4, \infty)$ **(B)** $(2, 5)$ only **(C)** $(5, \infty)$ only

(D) $(4, \infty)$ only **(E)** $(2, 4)$ only

(ii) Choose the interval(s) on which the *rate of change* is negative.

(A) $(-\infty, 2)$ and $(4, \infty)$ **(B)** $(2, 5)$ only **(C)** $(5, \infty)$ only

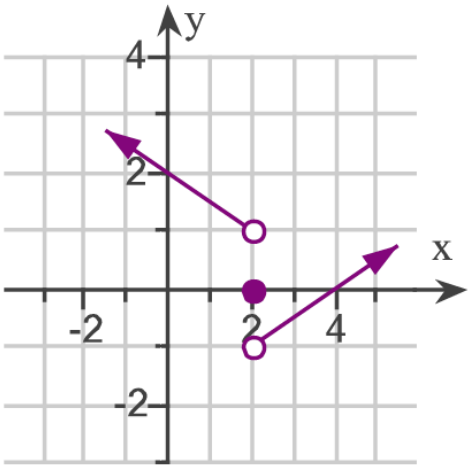
(D) $(4, \infty)$ only **(E)** $(2, 4)$ only

(iii) Choose the values at which the *rate of change* is 0.

(A) $x = 2$ only **(B)** $x = 2$ and $x = 4$ **(C)** $x = 2$ and $x = 5$

(D) $x = 5$ only **(E)** None of these

57. The graph of $g(x)$ is given below



Answer parts **(i)** through **(iii)**

(i) List the points in the graph in the interval $0 < x < 4$ at which the function is not differentiable.

(ii) Evaluate $g'(3)$.

(A) $g'(3) = -\frac{1}{2}$ **(B)** $g'(3) = 2$ **(C)** $g'(3) = \frac{3}{2}$

(D) $g'(3) = \frac{1}{2}$ **(E)** None of these

(iii) Find the value(s) of x such that $g'(x) = 0$.

58. Suppose the demand for a certain item is given by $D(p) = -3p^2 - 5p + 200$, where p represents the price of the item in dollars. Answer parts **(i)** and **(ii)**

(i) Find the rate of change of demand with respect to price. The rate of change with respect to price is

(A) $-6p - 5$ **(B)** $-3p^2 - 5p + 200$ **(C)** $-3p$

(D) $-6p + 195$ **(E)** none of these

(ii) The rate of change of demand when the price is \$11 is -71. Choose the correct interpretation below.

(A) When the price is \$11, demand is decreasing at a rate of about 71 items for each increase in price of \$11.

(B) When the price is \$11, demand is increasing at a rate of about 71 items for each increase in price of \$11.

(C) When the price is \$11, demand is increasing at a rate of about 71 items for each increase in price of \$1.

(D) When the price is \$11, demand is decreasing at a rate of about 71 items for each increase in price of \$1.

59. For $f(x) = x^2 + x$, find the equation of the tangent line when $x = -4$.
The tangent line is

(A) $y = -4(x - 7) + 12$ **(B)** $y = -7(x - 4) - 20$ **(C)** $y = -4(x + 7) - 20$

(D) $y = -7(x + 4) + 12$ **(E)** does not exist

60. The cost of recycling q tons of paper is given in the following table.

q (tons)	1000	1500	2000	2500	3000	3500
$C(q)$ (dollars)	2500	3200	3640	4060	4270	4415

Estimate the marginal cost at $q = 2000$.

(A) between \$0.00 per ton and \$1.00 per ton **(B)** between \$1.00 per ton and \$2.00 per ton

(C) between \$2.00 per ton and \$3.00 per ton **(D)** between \$3.00 per ton and \$4.00 per ton

(E) between \$4.00 per ton and \$5.00 per ton

61. Let $f(x) = x^2 - 2x$. We will work through the 4-step process for finding the derivative $f'(x)$. Answer parts **(i)** through **(v)**

(i) Evaluate $f(x+h)$.

(A) $f(x+h) = x^2 + h^2 - 2x - 2h$

(B) $f(x+h) = x^2 + xh + h^2 - 2x - 2h$

(C) $f(x+h) = x^2 + 2xh + h^2 - 2x - 2h$

(D) $f(x+h) = 2x + 2h - 2$

(E) $f(x+h) = x^2 - 2x + h$

(ii) Simplify $f(x+h) - f(x)$.

(A) $f(x+h) - f(x) = h$

(B) $f(x+h) - f(x) = 2xh + h^2 - 2h$

(C) $f(x+h) - f(x) = h^2 - 2h$

(D) $f(x+h) - f(x) = xh + h^2 - 2h$

(E) $f(x+h) - f(x) = 2h$

(iii) Simplify $\dfrac{f(x+h) - f(x)}{h}$.

(A) $\dfrac{f(x+h) - f(x)}{h} = 2x + h - 2$

(B) $\dfrac{f(x+h) - f(x)}{h} = 2xh - 2h + h^2$

(C) $\dfrac{f(x+h) - f(x)}{h} = x^2 + h - 2x - 2$

(D) $\dfrac{f(x+h) - f(x)}{h} = h - 2$

(E) $\dfrac{f(x+h) - f(x)}{h} = 2$

(iv) Evaluate $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

(A) $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = -2$

(B) $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = 0$

(C) $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = -2x - 2$

(D) $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = 2$

(E) $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = 2x - 2$

(v) What is $f'(x)$?

(A) $f'(x) = -2$

(B) $f'(x) = 0$

(C) $f'(x) = 2x - 2$

(D) $f'(x) = 2$

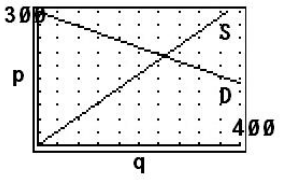
(E) $f'(x) = -2x - 2$

62. If $f(x) = x^2 + 3$, use the definition of the derivative to find $f'(x)$.

Formulas You Might Find Useful

$$
\begin{aligned}
I &= Prt \\
A &= P\left(1+\frac{r}{m}\right)^{mt} \\
A &= Pe^{rt} \\
r_E &= \left(1+\frac{r}{m}\right)^{m}-1 \\
r_E &= e^{r}-1 \\
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}
\end{aligned}
$$

Solutions to Study Guide - Part 1

1. B
2. B
3. A
4. D
5. A
6. **(i)** D
   (ii) World grain production increased at a rate of 26.9 million tons per year from 1975 to 2005.
   (iii) In late 2021
7. **(i)** B
   (ii) \$66.25
   (iii) It will cost \$66.25 to rent a snowboard for 5 hours.
8. C
9. **(i)** B
   (ii) A
   (iii) C
10. D
11. B
12. **(i)**


    (ii) \$200
13. A
14. **(i)** C
    (ii) B
    (iii) A
15. **(i)** B
    (ii) A
    (iii) $x = 2$, $x = 4$
16. **(i)** D
    (ii) A
17. **(i)** A
    (ii) A
    (iii) B
    (iv) D
    (v) B

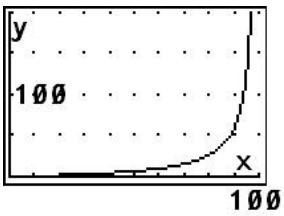
18. **(ii)** B
    (iii) C

19. **(i)** Total Revenue $= (900 + 50x)(60 - x)$
    (ii) 21
    (iii) \$76,050

20. **(i)** B
    (ii) C

21. B

22. **(i)** B
    (ii) D
    (iii) A

23. **(i)** A
    (ii) No
    (iii) 

24. A

25. C

26. B

27. E

28. **(i)** B
    (ii) C
    (iii) D

29. B

30. B

31. E

32. C

33. D

34. C

35. D

36. **(i)** 6% compounded quarterly
    (ii) $830.84
    (iii) 6% compounded quarterly: 6.14%, 5.75% compounded continuously: 5.92%
    (iv) 2.05 years

37. **(i)** 300 units
    (ii) 1.39 years
    (iii) No
    (iv) 1100 units

38. **(i)** B
    (ii) D

39. A

40. **(i)**

x	-1.1	-1.01	-1.001	-0.999	-0.99	-0.9
$\dfrac{x^2-1}{x+1}$	-2.1	-2.01	-2.001	-1.999	-1.99	-1.9

    (ii) A
    (iii) B

41. 10

42. B

43. E

44. D

45. E

46. D

47. A

48. **(i)** E
    (ii) C
    (iii) A

49. D

50. A

51. A

52. **(i)** D
    (ii) C
    (iii) E

53. C

54. B

55. B

56. **(i)** A
    (ii) E
    (iii) B

57. **(i)** $x = 2$
    (ii) D
    (iii) There are no values.

58. **(i)** A
    (ii) D

59. D

60. A

61. **(i)** C
    (ii) B
    (iii) A
    (iv) E
    (v) C

62.
$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{((x+h)^2+3)-(x^2+3)}{h} = \lim_{h\to 0} \frac{(x^2+2xh+h^2+3)-(x^2+3)}{h} \\ &= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0}(2x+h) = 2x \end{aligned}$$